Exercise 25

Prove the statement using the precise definition of a limit.

$$\lim_{x\to 2}(14-5x)=4$$

Solution

Proving this limit is logically equivalent to proving that

$$\text{if} \quad |x-2|<\delta \qquad \text{then} \qquad |(14-5x)-4|<\varepsilon$$

for all positive ε. Start by working backwards, looking for a number δ that's greater than $|x-2|$.

$$|(14-5x)-4|<\varepsilon$$

$$|10-5x|<\varepsilon$$

$$|-5(x-2)|<\varepsilon$$

$$5|x-2|<\varepsilon$$

$$|x-2|<\frac{\varepsilon}{5}$$

Choose $\delta=\varepsilon/5$. Now, assuming that $|x-2|<\delta$,

$$\begin{aligned}|(14-5x)-4| &= |10-5x| \\ &= |-5(x-2)| \\ &= 5|x-2| \\ &< 5\delta \\ &= 5\left(\frac{\varepsilon}{5}\right) \\ &= \varepsilon.\end{aligned}$$

Therefore, by the precise definition of a limit,

$$\lim_{x\to 2}(14-5x)=4.$$